Assignment 7

This homework is due Friday March 13.

There are total 50 points in this assignment. 45 points is considered 100%. If you go over 45 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge*. Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 4.3, 4.4, 5.1 of Textbook.

(1) [5pt] Use the ratio test to find the disk of convergence of the following series.

(a) $\sum_{n=0}^{\infty} (-1+i)^n z^n$.

(b) $\sum_{n=0}^{\infty} (-1+i)^n z^{2n}$.

(c) $\sum_{n=0}^{\infty} \frac{z^n}{(3-4i)^n}$.

(d) $\sum_{n=0}^{\infty} \frac{(z+i)^n}{(3-4i)^n}$.

(2) [10pt] Find radius of convergence of the following series.

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{z^n}{(2n)!}$.

(b) $\sum_{n=0}^{\infty} n! z^n$.

(c) $\sum_{n=0}^{\infty} n! z^{n!}$.

(d) $\sum_{n=0}^{\infty} \left(\frac{4n^2}{2n+1} - \frac{6n^2}{3n+4}\right) z^n$.

(e) $\sum_{n=0}^{\infty} (2-(-1)^n)^n z^n$.

(f) $\sum_{n=0}^{\infty} \frac{n(n-1)z^n}{(3+4i)^n}$.

(3) [10pt]

(a) Differentiate termwise the equality $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ twice.

(b) Show that $\sum_{n=0}^{\infty} (n+1)^2 z^n = \frac{1+z}{(1-z)^3}$. For what values of z is this equality valid?

(4) [5pt] Show that for $|z-i| < \sqrt{2}$, $\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}$.

(*Hint:* $\frac{1}{1-z} = \frac{1}{(1-i)-(z-i)} = \frac{1}{1-i}\left(\frac{1}{1-\frac{z-i}{1-i}}\right)$. Consider a geometric series with ratio $r = \frac{z-i}{1-i}$. In particular, when is $|r| < 1$?)

(5) [5pt] Express e^z in the form $u + iv$ for the following z.

(a) $-\frac{\pi}{3}$.

(b) $\frac{1}{2} - i\frac{\pi}{4}$.

(c) $-4+5i$.

(d) $\frac{\pi}{3} - 2i$.

(e) $-1 + i\frac{3\pi}{2}$.

(6) [5pt] Use the fact that e^{z^2} is analytic to show that $e^{x^2-y^2}\sin 2xy$ is harmonic.

(7) [10pt] Show the following concerning the exponential map.

(a) The image of the first quadrant $\{(x,y) : x > 0, y > 0\}$ is the region $\{w : |w| > 1\}$.

(b) If a is a real constant, the horizontal strip $\{(x,y) : a < y \leq a + 2\pi\}$ is mapped one-to-one and onto all nonzero complex numbers.

(c) The image of the vertical line segment $\{(x,y) : x = 2, y = t\}$, where $\frac{\pi}{6} < t < \frac{7\pi}{6}$, is half a circle.

(d) The image of the horizontal ray $\{(x,y) : x > 0, y = \frac{\pi}{3}\}$ is a ray.